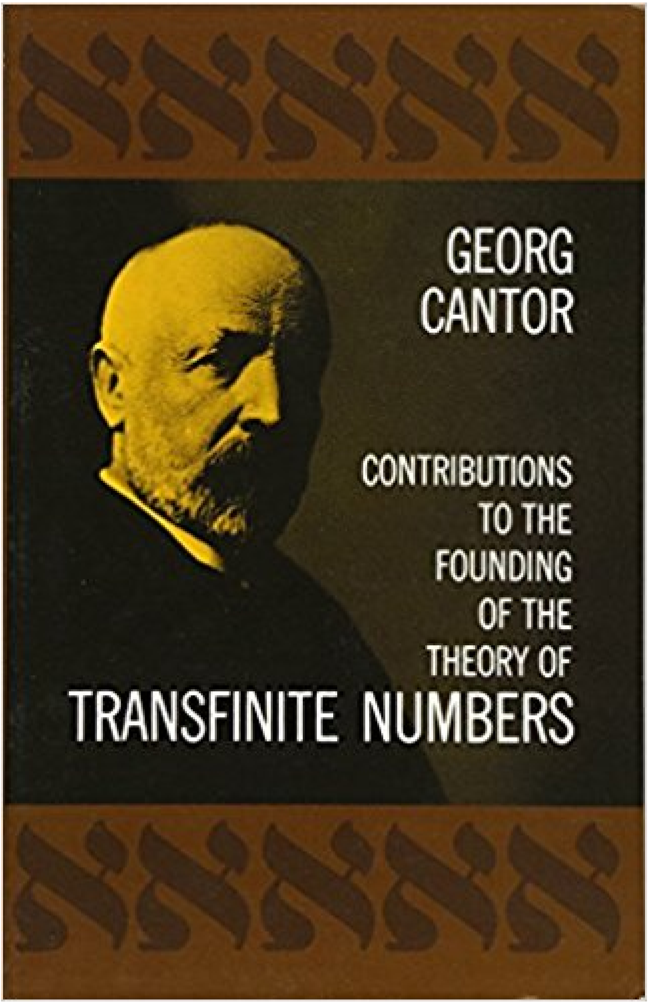
Contributions To The Founding Of The Theory Of Transfinite Numbers (Dover Books On Mathematics)



Synopsis

One of the greatest mathematical classics of all time, this work established a new field of mathematics which was to be of incalculable importance in topology, number theory, analysis, theory of functions, etc., as well as in the entire field of modern logic. It is rare that a theory of such fundamental mathematical importance is expressed so simply and clearly: the reader with a good grasp of college mathematics will be able to understand most of the basic ideas and many of the proofs.Cantor first develops the elementary definitions and operations of cardinal and ordinal numbers and analyzes the concepts of "canlinality" and "ordinality." He covers such topics as the addition, multiplication, and exponentiation of cardinal numbers, the smallest transfinite cardinal number, the ordinal types of simply ordered aggregates, operations on ordinal types, the ordinal type of the linear continuum, and others. He then develops a theory of well-ordered aggregates, and investigates the ordinal numbers of well-ordered aggregates and the properties and extent of the transfinite ordinal numbers.An 82-page introduction by the eminent mathematical historian Philip E. B. Jourdain first sketches the background of Cantor's theory, discussing the contributions of such predecessors as Veicrstrass, Cauchy, Dedekind, Dirichlet, Riemann, Fourier, and Hankel; it then traces the development of the theory by summarizing and analyzing Cantor's earlier work. A bibliographical note provides information on further investigations in the theory of transfinite numbers by Frege, Peano, Whitehead, Russell, etc."Would serve as well as any modern textto initiate a student in this exciting branch of mathematics." Ã¢â ¬â • Mathematical Gazette.

Book Information

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Customer Reviews

Dover has up ’till now done a flawless job in its Kindle transcriptions, but I had to return this particular book. The scans of Cantor’s equations and theorems are low quality blow-ups from the print edition and, what’s worse, when some are transcribed into actual text, they often get garbled into question marks. Perhaps it’s a technological problem and the kindle software just cannot parse some of the more esoteric mathematical symbols used in this publication. In any case, I advise everyone to stick with their hardcopies until Dover issues an improved edition.

Cantor probably contributed more to our modern theory than any other mathematician. In my opinion, a true genius who dedicated most of his life to getting a clearer and more in-depth understanding of the topic than almost any other human. However, reading this book is for those who might enjoy reading "Euclid’s Elements." A wonderful co-joining of a truly powerful mind and allowing insights into how they approached the topic, but definitely not a modern textbook. For a historian, it is where modern foundations of numbers were first treated. I hope you enjoy it as much as I did.

Georg Ferdinand Ludwig Philipp Cantor (1845-1918) was a German mathematician, best known as the inventor of set theory.The translator wrote in the Preface to this book, "This volume contains a translation of the two very important memoirs of Georg Cantor on transfinite numbers which appeared in the Mathematishe Annalen for 1895 and 1897 under the title, `BeitrÃƒÂ¤ge zur BegrÃƒÂ ndung der transfiniten Mengen-lehre.’... These memoirs are the final and logically purified statement of many of the most important results of the long series of memoirs begun by Cantor in 1870... It was in these researches that the need for the transfinite numbers first showed itself, and it is only by the study of these researches that the majority of us can annihilate the feeling of arbitrariness and even insecurity about the introduction of these numbers... The philosophical revolution brought about by Cantor’s work was even greater, perhaps, than the mathematical one... mathematicians joyfully accepted, built upon, scrutinized, and perfected the foundations of Cantor’s undying theory; but very many philosophers combated it. This seems to have been because very few understood it. I hope that this book may help to make the subject better known to both philosophers and mathematicians."He adds in the Introduction, "It is of the utmost importance to realize that, whereas until [Karl[Weierstrass’s time such subjects as the theory of points of condensation of an infinite aggregate and the theory of irrational numbers, on which the founding of the theory of functions depends, were hardly ever investigated... Weierstrass carried research into the principles of arithmetic farther than it had been carried before. But we must also realize that there were questions, such as the nature of whole number itself, to which he made no valuable contributions. These questions... were... historically the last to be dealt with. Before this could happen, arithmetic had to receive a development, by means of Cantor's discovery of transfinite numbers, into a theory of cardinal and ordinal numbers, both finite and transfinite, and logic had to be sharpened, as it was by Dedekind, Frege, Peano and Russell---to a great extent owing to the needs which this theory made evident." (Pg. 22-23)"All so-called proofs of the impossibility of actually infinite numbers," said Cantor, "are, as may be shown in every particular case and also on general grounds, false in that they begin by attributing to the numbers in question all the properties of finite numbers, whereas the infinite numbers, if they are to be thinkable in any form, must constitute quite a new kind of number as opposed to the finite numbers, and the nature of this new kind of number is dependent on the nature of things and is an object of investigation, but not of our arbitrariness or our prejudice." (Pg. 74)The translator adds, "When Cantor said that he had solved the chief part of the problem of determining the various powers in nature, he meant that he had almost proved that the power of the arithmetical continuum is the same as the power of the ordinal numbers of the second class. In spite of the fact that Cantor firmly believed this, possibly on account of the fact that all known aggregates in the continuum had been found to be either of the first power or of the power of the continuum, the proof or disproof of this theorem has not even now been carried out, and there is come ground for believing that it cannot be carried out." (Pg. 76)He adds in the Notes at the end of the book, "Although Frege worked out, in the first volume of hisÃ Â The Basic Laws of Arithmetic, an important part of arithmetic, with a logical accuracy previously unknown and for years afterward almost unknown, his ideas did not become at all widely known until Bertrand Russell... gave prominence to them in hisÃ Â The Principles of MathematicsÃ Â of 1903. The two chief reasons in favour of this definition are that it avoids, by a construction of `numbers' out of the fundamental entities of logic, the assumption that there are certain new and undefined entities called `numbers'; and that it allows us to deduce at once that the class defined is not empty, so that the cardinal number of u `exists' in the sense defined in logic: in fact, since u is equivalent to itself, the cardinal number of u has u at least as a member. Russell also gave an analogous definition for ordinal types of the more general `relation numbers.'" (Pg. 202-203)If set theory is "your thing" (particularly the historical development of it), you will appreciate (and perhaps even treasure!) this book. More "casual" readers should probably avoid it, however.

Georg Cantor's final and logically purified memoir on transfinite numbers was published in the late

1890s. This Dover reprint is the 1915 English translation by the mathematician Philip E. B. Jourdain; it also includes a lengthy, technically diffcult introduction by Jourdain.Contributions to the Founding of the Theory of Transfinite Numbers is not suitable as an introduction. I unwisely disregarded caution from an earlier reviewer that Cantor's work would not be appropriate for a beginner in set theory. (I thought that I was reasonably acquainted with set theory, but I do admit that I was not a math major.)The 82-page introduction by Jourdain assumes that the reader is reasonably familiar with the work of key nineteenth century mathematicians. While it is possible to skip the introduction, Jourdain's context setting is quite helpful. Cantor's transfinite numbers are so innovative and so unexpected that it almost seems as though they spring forth in a vacuum, but Jourdain shows that the earlier work of Dirichlet, Cauchy, Riemann, and Weierstrass helped point the way for Cantor.Cantor's memoir (that is, his two-part discussion of transfinite number theory) comprise the remaining 125 pages. The difficulty with Cantor's axiomatic presentation is two-fold. First, the material itself is not easy - despite Cantor's careful approach. I even bogged down for awhile on his early discussion of the exponentiation of powers and how this leads to aleph-zero. And second, much of his terminology is outdated and unfamiliar. For example, there is no mention of sets, just aggregates and parts. Another example is that Cantor speaks of reciprocal and univocal correspondence. I have yet to complete Cantor's work, but I am continuing to plod along.A recommendation: A much better starting point for readers new to transfinite numbers is a fascinating book by Mary Tiles, titled The Philosophy of Set Theory - An Historical Introduction to Cantor's Paradise. This work targets mathematics and philosophy majors, but is accessible to others.

There's nothing like reading the original. Here is the abstract theory of transfinite ordinals described by its originator, Georg Cantor.It's probably not the best introduction to set theory for a beginner. The book focuses more on ordinal numbers than on cardinals or general sets. It's not a great reference, either, since so many important results in set theory have been proven in the 100 years since Cantor. But I like this book a lot nonetheless. The exposition is beautiful -- concise, clear, and logical. It's one of the most nicely presented math books I've read.

Contributions to the Founding of the Theory of Transfinite Numbers (Dover Books on Mathematics) Contributions to the Founding of the Theory of Transfinite Numbers Elementary Theory of Numbers (Dover Books on Mathematics) Elementary Number Theory: Second Edition (Dover Books on Mathematics) 2nd (second) Edition by Underwood Dudley published by Dover Publications (2008) Fractals, Wavelets, and their Applications: Contributions from the International Conference and

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